

AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions and listings of claims in the application:

Claims 1-17 are cancelled.

18. (Previously Presented) A system of computing and rendering the nature of bound atomic and atomic ionic electrons from physical solutions of the charge, mass, and current density functions of atoms and atomic ions, which solutions are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration, comprising:

a processor for processing and solving the equations for charge, mass, and current density functions of electron(s) in a selected atom or ion, wherein the equations are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration; and

a display in communication with the processor for displaying the current and charge density representation of the electron(s) of the selected atom or ion; wherein the physical, Maxwellian solutions of the charge, mass, and current density functions of atoms and atomic ions comprises a solution of the classical wave equation

$$\left[\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right] \rho(r, \theta, \phi, t) = 0$$

19. (Original) The system of claim 18, wherein the time, radial, and angular solutions of the wave equation are separable.

20. (Original) The system of claim 18, wherein the boundary constraint of the wave equation solution is nonradiation according to Maxwell's equations.

21. (Original) The system of claim 20, wherein a radial function that satisfies the boundary condition is a radial delta function

$$f(r) = \frac{1}{r^2} \delta(r - r_n)$$

22. (Original) The system of claim 21, wherein the boundary condition is met for a time harmonic function when the relationship between an allowed radius and the electron wavelength is given by

$$2\pi r_n = \lambda_n,$$

$$\omega = \frac{\hbar}{m_e r^2}, \text{ and}$$

$$v = \frac{\hbar}{m_e r}$$

where ω is the angular velocity of each point on the electron surface, v is the velocity of each point on the electron surface, and r is the radius of the electron.

23. (Original) The system of claim 22, wherein the spin function is given by

the uniform function $Y_0^0(\phi, \theta)$ comprising angular momentum components of $L_{xy} = \frac{\hbar}{4}$ and

$$L_z = \frac{\hbar}{2}.$$

24. (Original) The system of claim 23, wherein the atomic and atomic ionic charge and current density functions of bound electrons are described by a charge-density (mass-density) function which is the product of a radial delta function, two angular functions (spherical harmonic functions), and a time harmonic function:

$$\rho(r, \theta, \phi, t) = f(r) A(\theta, \phi, t) = \frac{1}{r^2} \delta(r - r_n) A(\theta, \phi, t); \quad A(\theta, \phi, t) = Y(\theta, \phi) k(t)$$

wherein the spherical harmonic functions correspond to a traveling charge density wave confined to the spherical shell which gives rise to the phenomenon of orbital angular momentum.

25. (Original) The system of claim 24, wherein based on the radial solution, the angular charge and current-density functions of the electron, must be a solution of the wave equation in two dimensions (plus time),

$$\left[\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right] A(\theta, \phi, t) = 0$$

where

$$\rho(r, \theta, \phi, t) = f(r)A(\theta, \phi, t) = \frac{1}{r^2} \delta(r - r_s)A(\theta, \phi, t) \quad \text{and} \quad A(\theta, \phi, t) = Y(\theta, \phi)k(t)$$

$$\left[\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right)_{r, \phi} + \frac{1}{r^2 \sin^2 \theta} \left(\frac{\partial^2}{\partial \phi^2} \right)_{r, \theta} - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right] A(\theta, \phi, t) = 0$$

where v is the linear velocity of the electron.

26. (Original) The system of claim 25, wherein the charge-density functions including the time-function factor are

$$\lambda = 0$$

$$\rho(r, \theta, \phi, t) = \frac{e}{8\pi r^2} [\delta(r - r_s)] [Y_0^0(\theta, \phi) + Y_\ell^m(\theta, \phi)]$$

$$\lambda \neq 0$$

$$\rho(r, \theta, \phi, t) = \frac{e}{4\pi r^2} [\delta(r - r_s)] [Y_0^0(\theta, \phi) + \operatorname{Re} \{ Y_\ell^m(\theta, \phi) e^{i\omega_r t} \}]$$

where $Y_\ell^m(\theta, \phi)$ are the spherical harmonic functions that spin about the z-axis with

angular frequency ω_n with $Y_0^0(\theta, \phi)$ the constant function

$$\operatorname{Re} \left\{ Y_\ell^m(\theta, \phi) e^{i\omega_n t} \right\} = P_\ell^m(\cos \theta) \cos(m\phi + \omega_n t) \quad \text{where to keep the form of the spherical}$$

harmonic as a traveling wave about the z-axis, $\omega_n' = m\omega_n$.

27. (Original) The system of claim 26, wherein the spin and angular moment of inertia, I, angular momentum, L, and energy, E, for quantum number ℓ are given by

$$\lambda = 0$$

$$I_z = I_{spin} = \frac{m_e r_n^2}{2}$$

$$L_z = I\omega \mathbf{i}_z = \pm \frac{\hbar}{2}$$

$$E_{rotational} = E_{rotational, spin} = \frac{1}{2} \left[I_{spin} \left(\frac{\hbar}{m_e r_n^2} \right)^2 \right] = \frac{1}{2} \left[\frac{m_e r_n^2}{2} \left(\frac{\hbar}{m_e r_n^2} \right)^2 \right] = \frac{1}{4} \left[\frac{\hbar^2}{2I_{spin}} \right]$$

$$\lambda \neq 0$$

$$I_{orbital} = m_e r_n^2 \left[\frac{\ell(\ell+1)}{\ell^2 + \ell + 1} \right]^{\frac{1}{2}}$$

$$L_z = m\hbar$$

$$L_z \text{ total} = L_{z, spin} + L_{z, orbital}$$

$$E_{rotational, orbital} = \frac{\hbar^2}{2I} \left[\frac{\ell(\ell+1)}{\ell^2 + 2\ell + 1} \right]$$

$$T = \frac{\hbar^2}{2m_e r_n^2}$$

$$\langle E_{rotational, orbital} \rangle = 0.$$

28. (Previously Presented) The system of claim 18, wherein the force balance equation for one-electron atoms and ions is

$$\frac{m_e}{4\pi r_1^2} \frac{v_1^2}{r_1} = \frac{e}{4\pi r_1^2} \frac{Ze}{4\pi\epsilon_0 r_1^2} - \frac{1}{4\pi r_1^2} \frac{\hbar^2}{m_p r_n^3}$$

$$r_1 = \frac{a_H}{Z}$$

where a_H is the radius of the hydrogen atom.

29. (Original) The system of claim 28, wherein from Maxwell's equations, the potential energy V , kinetic energy T , electric energy or binding energy E_{ele} are

$$V = \frac{-Ze^2}{4\pi\epsilon_0 r_1} = \frac{-Z^2 e^2}{4\pi\epsilon_0 a_H} = -Z^2 \times 4.3675 \times 10^{-18} J = -Z^2 \times 27.2 eV$$

$$T = \frac{Z^2 e^2}{8\pi\epsilon_0 a_H} = Z^2 \times 13.59 eV$$

$$T = E_{ele} = -\frac{1}{2} \epsilon_0 \int_{\infty}^{r_1} \mathbf{E}^2 dV \quad \text{where } \mathbf{E} = -\frac{Ze}{4\pi\epsilon_0 r^2}$$

$$E_{ele} = -\frac{Z^2 e^2}{8\pi\epsilon_0 a_H} = -Z^2 \times 2.1786 \times 10^{-18} J = -Z^2 \times 13.598 eV$$

30. (Previously Presented) The system of claim 18, wherein the force balance equation solution of two electron atoms is a central force balance equation with the nonradiation condition given by

$$\frac{m_e}{4\pi r_2^2} \frac{v_2^2}{r_2} = \frac{e}{4\pi r_2^2} \frac{(Z-1)e}{4\pi\epsilon_0 r_2^2} + \frac{1}{4\pi r_2^2} \frac{\hbar^2}{Z m_e r_2^3} \sqrt{s(s+1)}$$

which gives the radius of both electrons as

$$r_2 = r_1 = a_0 \left(\frac{1}{Z-1} - \frac{\sqrt{s(s+1)}}{Z(Z-1)} \right); s = \frac{1}{2}$$

31. (Original) The system of claim 30, wherein the ionization energy for helium, which has no electric field beyond r_1 is given by

$$\text{Ionization Energy}(He) = -E(\text{electric}) + E(\text{magnetic})$$

where,

$$E(\text{electric}) = -\frac{(Z-1)e^2}{8\pi\epsilon_0 r_i}$$

$$E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r_i^3}$$

For $3 \leq Z$

$$\text{Ionization Energy} = -\text{Electric Energy} - \frac{1}{Z} \text{Magnetic Energy}$$

32. (Previously Presented) The system of claim 18, wherein the electrons of multielectron atoms all exist as orbitspheres of discrete radii which are given by r_n of the radial Dirac delta function, $\delta(r - r_n)$.

33. (Original) The system of claim 32, wherein electron orbitspheres may be spin paired or unpaired depending on the force balance which applies to each electron wherein the electron configuration is a minimum of energy.

34. (Original) The system of claim 33, wherein the minimum energy configurations are given by solutions to Laplace's equation.

35. (Original) The system of claim 34, wherein the electrons of an atom with the same principal and ℓ quantum numbers align parallel until each of the m_ℓ levels are occupied, and then pairing occurs until each of the m_ℓ levels contain paired electrons.

36. (Original) The system of claim 35, wherein the electron configuration for one through twenty-electron atoms that achieves an energy minimum is: $1s < 2s < 2p < 3s < 3p < 4s$.

37. (Original) The system of claim 36, wherein the corresponding force balance of the central centrifical, Coulombic, paramagnetic, magnetic, and diamagnetic forces for an electron configuration was derived for each n-electron atom that was solved for the radius of each electron.

38. (Original) The system of claim 37, wherein the central Coulombic force is that of a point charge at the origin since the electron charge-density functions are spherically symmetrical with a time dependence that is nonradiative.

39. (Original) The system of claim 38, wherein the ionization energies are obtained using the calculated radii in the determination of the Coulombic and any magnetic energies.

40. (Original) The system of claim 39, wherein the general equation for the radii of s electrons is given by

$$r_n = \frac{a_0 \left(1 + (C - D) \frac{\sqrt{3}}{2Z} \right)}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)^{\pm a_0}} \frac{1}{2} \left[\frac{\left(1 + (C - D) \frac{\sqrt{3}}{2Z} \right)^2}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} \right. \\ \left. + \frac{20\sqrt{3} \left[\left[\frac{Z - n}{Z - (n-1)} \right] Er_m \right]}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} \right]$$

r_m in units of a₀

where positive root must be taken in order that r_n > 0;

Z is the nuclear charge, n is the number of electrons,

r_m is the radius of the proceeding filled shell(s) given by

$$r_m = \frac{a_0 \left(1 + (C - D) \frac{\sqrt{3}}{2Z} \right)}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)^{\pm a_0}} \frac{1}{2} \left[\frac{\left(1 + (C - D) \frac{\sqrt{3}}{2Z} \right)^2}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} \right. \\ \left. + \frac{20\sqrt{3} \left[\left[\frac{Z - n}{Z - (n-1)} \right] Er_m \right]}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} \right]$$

r_m in units of a₀

for the preceding s shell(s);

$$r_n = \frac{\frac{a_0}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)^{\pm a_0}} + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 \right)}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)^2}}{2}$$

r_3 in units of a_0

for the 2p shell, and

$$r_n = \frac{\frac{a_0}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)^{\pm a_0}} + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2} \right) r_{12} \right)}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)^2}}{2}$$

r_{12} in units of a_0

for the 3p shell;

the parameter A corresponds to the diamagnetic force, $F_{diamagnetic}$:

$$\mathbf{F}_{diamagnetic} = -\frac{\hbar^2}{4m_e r_3^2 r_i} \sqrt{s(s+1)} \mathbf{i}_r$$

the parameter B corresponds to the paramagnetic force, F_{mag2} :

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_1^2} \sqrt{s(s+1)} \mathbf{i}_r$$

the parameter C corresponds to the diamagnetic force, $F_{diamagnetic3}$

$$\mathbf{F}_{diamagnetic\ 3} = -\frac{1}{Z} \frac{8\hbar^2}{m_e r_{11}^3} \sqrt{s(s+1)} \mathbf{i}_r$$

the parameter D corresponds to the paramagnetic force, F_{mag} :

$$\mathbf{F}_{mag} = \frac{1}{4\pi r_2^2} \frac{1}{Z} \frac{\hbar^2}{m_e r^3} \sqrt{s(s+1)}, \text{ and}$$

the parameter E corresponds to the diamagnetic force, $F_{diamagnetic2}$, due to a relativistic effect with an electric field for $r > r_n$:

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-3}{Z-2}\right] \frac{r\hbar^2}{m_e r_3^4} 10\sqrt{3/4} \mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-11}{Z-10}\right] \left(1 + \frac{\sqrt{2}}{2}\right) \frac{r_{10}\hbar^2}{m_e r_{11}^4} 10\sqrt{s(s+1)} \mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-n}{Z-(n-1)}\right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{2}}{2} + \frac{1}{2}\right) \frac{r_{18}\hbar^2}{m_e r_n^4} 10\sqrt{s(s+1)} \mathbf{i}_r$$

wherein the parameters of atoms filling the 1s, 2s, 3s, and 4s orbitals are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of s Electrons (s state)	Diamag Force Factor A	g. Force Factor B	Diamag. Force Factor C	Para mag. Force Factor D	Diamag. Force Factor E
Neutral $1s^1$ 1 e Atom <i>H</i>		$^2S_{1/2}$	$\begin{array}{c} \uparrow \\ 1s \end{array}$	0	0	0	0	0
Neutral $1s^2$ 2 e Atom <i>He</i>		1S_0	$\begin{array}{c} \uparrow \downarrow \\ 1s \end{array}$	0	0	0	1	0
Neutral $2s^1$ 3 e Atom <i>Li</i>		$^2S_{1/2}$	$\begin{array}{c} \uparrow \\ 2s \end{array}$	1	0	0	0	0
Neutral $2s^2$ 4 e Atom <i>Be</i>		1S_0	$\begin{array}{c} \uparrow \downarrow \\ 2s \end{array}$	1	0	0	1	0
Neutral $1s^2 2s^2 2p^6 3s^1$ 11 e Atom <i>Na</i>		$^2S_{1/2}$	$\begin{array}{c} \uparrow \\ 3s \end{array}$	1	0	8	0	0
Neutral $1s^2 2s^2 2p^6 3s^2$ 12 e Atom <i>Mg</i>		1S_0	$\begin{array}{c} \uparrow \downarrow \\ 3s \end{array}$	1	3	12	1	0
Neutral $1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$ 19 e Atom <i>K</i>		$^2S_{1/2}$	$\begin{array}{c} \uparrow \\ 4s \end{array}$	2	0	12	0	0
Neutral $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$ 20 e Atom <i>Ca</i>		1S_0	$\begin{array}{c} \uparrow \downarrow \\ 4s \end{array}$	1	3	24	1	0
1 e ion $1s^1$		$^2S_{1/2}$	$\begin{array}{c} \uparrow \\ 1s \end{array}$	0	0	0	0	0
2 e ion $1s^2$		1S_0	$\begin{array}{c} \uparrow \downarrow \\ 1s \end{array}$	0	0	0	1	0
3 e ion $2s^1$		$^2S_{1/2}$	$\begin{array}{c} \uparrow \\ 2s \end{array}$	1	0	0	0	1
4 e ion $2s^2$		1S_0	$\begin{array}{c} \uparrow \downarrow \\ 2s \end{array}$	1	0	0	1	1

11 e Ion	$1s^2 2s^2 2p^6 3s^1$	$^2S_{1/2}$	$\frac{\uparrow}{3s}$	1	4	8	0	$1 + \frac{\sqrt{2}}{2}$
12 e Ion	$1s^2 2s^2 2p^6 3s^2$	1S_0	$\frac{\uparrow\downarrow}{3s}$	1	6	0	0	$1 + \frac{\sqrt{2}}{2}$
19 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$	$^2S_{1/2}$	$\frac{\uparrow}{4s}$	3	0	24	0	$2 - \sqrt{2}$
20 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$	1S_0	$\frac{\uparrow\downarrow}{4s}$	2	0	24	0	$2 - \sqrt{2}$

41. (Original) The system of claim 40, with the radii, r_n , wherein the ionization energy for atoms having an outer s-shell are given by the negative of the electric energy, $E(electric)$, given by:

$$E(Ionization) = -Electric Energy = \frac{(Z - (n-1))e^2}{8\pi\epsilon_0 r_n}$$

except that minor corrections due to the magnetic energy must be included in cases wherein the s electron does not couple to p electrons as given by

$$Ionization Energy(He) = -E(electric) + E(magnetic) \left(1 - \frac{1}{2} \left(\left(\frac{2}{3} \cos \frac{\pi}{3} \right)^2 + \alpha \right) \right)$$

$$Ionization Energy = -Electric Energy - \frac{1}{Z} Magnetic Energy$$

$$E(Ionization; Li) = \frac{(Z-2)e^2}{8\pi\epsilon_0 r_3} + \Delta E_{mag}$$

$$= 5.3178 \text{ eV} + 0.0860 \text{ eV} = 5.4038 \text{ eV}$$

$$E(Ionization) = E(Electric) + E_T$$

$$E(Ionization; Be) = \frac{(Z-3)e^2}{8\pi\epsilon_0 r_4} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r_4^3} + \Delta E_{mag}$$

, and

$$= 8.9216 \text{ eV} + 0.03226 \text{ eV} + 0.33040 \text{ eV} = 9.28430 \text{ eV}$$

$$E(Ionization) = -Electric Energy - \frac{1}{Z} Magnetic Energy - E_T.$$

42. (Original) The system of claim 41, wherein the radii and energies of the 2p electrons are solved using the forces given by

$$\mathbf{F}_{elec} = \frac{(Z-n)e^2}{4\pi\epsilon_0 r_e^2} \mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!} \frac{\hbar^2}{4m_e r_e^2 r_i} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 1} = \frac{1}{Z} \frac{\hbar^2}{m_e r_e^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_e^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 3} = \frac{1}{Z} \frac{\hbar^2}{m_e r_e^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_i \hbar^2}{m_e r_e^4} 10 \sqrt{s(s+1)} \mathbf{i}_r$$

and the radii r_3 are given by

$$r_1 = r_2 = \frac{\pm a_s \sqrt{\left(\frac{Z-3}{Z-2} \right) r_i 10 \sqrt{\frac{3}{4}}} + 4 \left(\frac{Z-3}{Z-2} \right) r_i \sqrt{\frac{3}{4}}}{2}$$

$$\left(\frac{Z-3}{Z-2} \right) r_i \sqrt{\frac{3}{4}}$$

$$\left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)$$

$$a_s \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)$$

r_1 in units of a_s

43. (Original) The system of claim 42, wherein the electric energy given by

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z-(n-1))e^2}{8\pi\epsilon_0 r_n}$$

gives the corresponding ionization energies.

44. (Original) The system of claim 43, wherein for each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^{n-4}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_s \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right]$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\pm a_0 \sqrt{\left(\frac{Z-3}{(Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1}} \right)^2 + 4 \left(\frac{[Z-3]r_1}{Z-2} \right)^2}}{2}$$

$$\left(\frac{a_0 \left(1 - \frac{\sqrt{3}}{4} \right)}{(Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1}} \right)$$

r_1 in units of a_0

and n-4 electrons in an orbitsphere with radius r_n given by

$$r_n = \frac{\pm a_0 \sqrt{\left(\frac{1}{(Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3}} \right)^2 + 4 \left(\frac{20\sqrt{3} \left(\frac{Z-n}{Z-(n-1)} \right) \left(1 - \frac{\sqrt{2}}{2} \right) r_3}{(Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3}} \right)^2}}{2}$$

r_3 in units of a_0

the positive root must be taken in order that $r_n > 0$;

the parameter A corresponds to the diamagnetic force, $F_{diamagnetic}$:

$$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell + |m|)!}{(2\ell + 1)(\ell - |m|)!} \frac{\hbar^2}{4m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r;$$

and the parameter B corresponds to the paramagnetic force, F_{mag2} :

$$\mathbf{F}_{mag2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r,$$

$$\mathbf{F}_{mag2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_1} \sqrt{s(s+1)} \mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{mag2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

wherein the parameters of five through ten-electron atoms are

Atom Type	Electron Configuration n	Ground State Term	Orbital Arrangement of 2p Electrons (2p state)	Diamagnetic Force Factor A	Paramagnetic Force Factor B
Neutral 5 e Atom B	$1s^2 2s^2 2p^1$	${}^2P_{1/2}^0$	$\begin{array}{ccc} \uparrow & _ & _ \\ 1 & 0 & -1 \end{array}$	2	0
Neutral 6 e Atom C	$1s^2 2s^2 2p^2$	3P_0	$\begin{array}{ccc} \uparrow & \uparrow & _ \\ 1 & 0 & -1 \end{array}$	$\frac{2}{3}$	0
Neutral 7 e Atom N	$1s^2 2s^2 2p^3$	${}^4S_{3/2}^0$	$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{1}{3}$	1
Neutral 8 e Atom O	$1s^2 2s^2 2p^4$	3P_2	$\begin{array}{ccc} \uparrow \downarrow & \uparrow & \uparrow \\ 1 & 0 & -1 \end{array}$	1	2
Neutral 9 e Atom F	$1s^2 2s^2 2p^5$	${}^2P_{3/2}^0$	$\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{2}{3}$	3
Neutral 10 e Atom Ne	$1s^2 2s^2 2p^6$	1S_0	$\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \downarrow \\ 1 & 0 & -1 \end{array}$	0	3
5 e Ion	$1s^2 2s^2 2p^1$	${}^2P_{1/2}^0$	$\begin{array}{ccc} \uparrow & _ & _ \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$	1
6 e Ion	$1s^2 2s^2 2p^2$	3P_0	$\begin{array}{ccc} \uparrow & \uparrow & _ \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$	4
7 e Ion	$1s^2 2s^2 2p^3$	${}^4S_{3/2}^0$	$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$	6
8 e Ion	$1s^2 2s^2 2p^4$	3P_2	$\begin{array}{ccc} \uparrow \downarrow & \uparrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$	6
9 e Ion	$1s^2 2s^2 2p^5$	${}^2P_{3/2}^0$	$\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$	9
10 e Ion	$1s^2 2s^2 2p^6$	1S_0	$\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \downarrow \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$	12

45. (Original) The system of claim 44, wherein the ionization energy for the boron atom is given by

$$\begin{aligned} E(\text{ionization}; B) &= \frac{(Z-4)e^2}{8\pi\epsilon_0 r_s} + \Delta E_{\text{mag}} \\ &= 8.147170901 \text{ eV} + 0.15548501 \text{ eV} = 8.30265592 \text{ eV} \end{aligned}$$

46. (Original) The system of claim 44, wherein the ionization energies for the n-electron atoms having the radii, r_n , are given by the negative of the electric energy, $E(\text{electric})$, given by

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z-(n-1))e^2}{8\pi\epsilon_0 r_n}$$

47. (Previously Presented) The system of claim 18, wherein the radii of the 3p electrons are given using the forces given by:

$$\begin{aligned} \mathbf{F}_{\text{elec}} &= \frac{(Z-n)e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r \\ \mathbf{F}_{\text{diamagnetic}} &= -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!} \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{\text{diamagnetic}} &= -\left(\frac{2}{3} + \frac{2}{3} + \frac{1}{3}\right) \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = -\left(\frac{5}{3}\right) \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{\text{mag}_2} &= \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{\text{mag}_2} &= (4+4+4) \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = \frac{1}{Z} \frac{12\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{\text{mag}_2} &= \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{\text{mag}_2} &= \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{\text{mag}_2} &= \frac{1}{Z} \frac{8\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \end{aligned}$$

and the radii r_{12} are given by

$$r_{12} = \frac{\frac{a_0}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)}^{\pm a_0} \left[\begin{array}{l} \left(\frac{1}{(Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}}} \right)^2 \\ + 20\sqrt{3} \left(\frac{Z-12}{Z-11} \right) \left(1 + \frac{\sqrt{2}}{2} \right) r_{10} \\ + \left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right) \end{array} \right]}{2} \quad .$$

r₁₀ in units of a₀

48. (Original) The system of claim 47, wherein the ionization energies are given by electric energy given by:

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z-(n-1))e^2}{8\pi\varepsilon_0 r_n}$$

49. (Previously Presented) The system of claim 18, wherein for each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^6 3s^2 3p^{n-12}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_p \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right]$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\left(a_0 \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right) \right)}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)} \pm a_0 \sqrt{\frac{\left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)^2 + 4 \left(\frac{[Z-3]}{[Z-2]} r_1 10 \sqrt{\frac{3}{4}} \right)}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2}}$$

r_1 in units of a_0

three sets of paired indistinguishable electrons in an orbitsphere with radius r_{10} given by:

$$r_{10} = \frac{\left(a_0 \left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_1} \right) \right)^{\pm a_0} \sqrt{\frac{1}{\left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_1} \right)^2} + \frac{20\sqrt{3} \left(\left[\frac{Z-10}{Z-9} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_1 \right)}{\left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_1} \right)}}}{2}$$

r_1 in units of a_0

two indistinguishable spin-paired electrons in an orbitsphere with radius r_{12} given by:

$$r_{12} = \frac{\frac{a_0}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)} \right)^2 + \frac{20\sqrt{3} \left(\left[\frac{Z-12}{Z-11} \right] \left(1 + \frac{\sqrt{2}}{2} \right) r_{10} \right)}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)}}}{2}$$

r₁₀ in units of a₀

and n-12 electrons in a 3p orbitsphere with radius r_n given by

$$r_n = \frac{\frac{a_0}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)} \right)^2 + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2} \right) r_{12} \right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)}}}{2}$$

r₁₂ in units of a₀

where the positive root must be taken in order that r_n > 0;

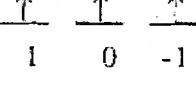
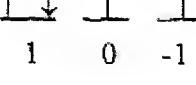
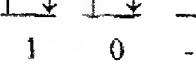
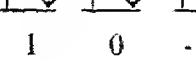
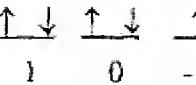
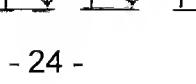
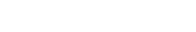
the parameter A corresponds to the diamagnetic force, F_{diamagnetic}:

$$\mathbf{F}_{diamagnetic} = - \sum_m \frac{(\ell + |m|)}{(2\ell + 1)(\ell - |m|)} \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

and the parameter B corresponds to the paramagnetic force, F_{mag2}:

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$
$$\mathbf{F}_{mag\ 2} = (4 + 4 + 4) \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = \frac{1}{Z} \frac{12\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$
$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$
$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r, \text{ and}$$
$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{8\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

wherein the parameters of thirteen through eighteen-electron atoms are

Atom Type	Electron Configuration	Ground State Orbital Term of 3p Electrons (3p state)	Diamagnetic Force Factor A	Paramagnetic Force Factor B
Neutral 13 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^1$	${}^2P_{1/2}^0$ 	$\frac{11}{3}$	0
<i>Al</i>				
Neutral 14 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^2$	3P_0 	$\frac{7}{3}$	0
<i>Si</i>				
Neutral 15 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^3$	${}^4S_{3/2}^0$ 	$\frac{5}{3}$	2
<i>P</i>				
Neutral 16 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^4$	3P_2 	$\frac{4}{3}$	1
<i>S</i>				
Neutral 17 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^5$	${}^2P_{3/2}^0$ 	$\frac{2}{3}$	2
<i>Cl</i>				
Neutral 18 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^6$	1S_0 	$\frac{1}{3}$	4
<i>Ar</i>				
13 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^1$	${}^2P_{1/2}^0$ 	$\frac{5}{3}$	12
14 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^2$	3P_0 	$\frac{1}{3}$	16
15 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^3$	${}^4S_{3/2}^0$ 	0	24
16 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^4$	3P_2 	$\frac{1}{3}$	24
17 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^5$	${}^2P_{3/2}^0$ 	$\frac{2}{3}$	32
18 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6$	1S_0 	∞	∞

50. (Original) The system of claim 49, wherein the ionization energies for the n-electron 3p atoms are given by electric energy given by:

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z - (n - 1))e^2}{8\pi\epsilon_0 r_n}$$

51. (Original) The system of claim 50, wherein the ionization energy for the aluminum atom is given by

$$\begin{aligned} E(\text{ionization; Al}) &= \frac{(Z - 12)e^2}{8\pi\epsilon_0 r_{13}} + \Delta E_{\text{mag}} \\ &= 5.95270 \text{ eV} + 0.031315 \text{ eV} = 5.98402 \text{ eV} \end{aligned}$$

52. (Cancelled)

53. (Cancelled)

54. (Cancelled)

55. (Currently Amended) A method comprising:

- a) inputting electron functions that are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration;
- b) inputting a trial electron configuration;
- c) inputting the corresponding centrifugal, Coulombic, diamagnetic and paramagnetic forces,
- d) forming the force balance equation comprising the centrifugal force equal to the sum of the Coulombic, diamagnetic and paramagnetic forces;
- e) solving the force balance equation for the electron radii;
- f) calculating the energy of the electrons using the radii and the corresponding electric and magnetic energies;

g) repeating Steps a-f for all possible electron configurations, and
h) outputting the lowest energy configuration and the corresponding electron
radii for that configuration,
wherein the output is rendered using electron functions given by at least one of the
group comprising:

$$\lambda = 0$$

$$\rho(r,\theta,\phi,t) = \frac{e}{8\pi r^2} [\delta(r - r_n)] [Y_0^0(\theta, \phi) + Y_\ell^m(\theta, \phi)]$$

$$\lambda \neq 0$$

$$\rho(r,\theta,\phi,t) = \frac{e}{4\pi r^2} [\delta(r - r_n)] [Y_0^0(\theta, \phi) + \operatorname{Re}\{Y_\ell^m(\theta, \phi)e^{i\omega_n t}\}]$$

where $Y_\ell^m(\theta, \phi)$ are the spherical harmonic functions that spin about the z-axis with
angular frequency ω_n with $Y_0^0(\theta, \phi)$ the constant function[[.]],

$$\operatorname{Re}\{Y_\ell^m(\theta, \phi)e^{i\omega_n t}\} = P_\ell^m(\cos \theta) \cos(m\phi + \omega_n t)$$

where to keep the form of the spherical harmonic as a traveling wave about the z-axis,
 $\omega_n' = m\omega_n$.

56. (Original) The method of claim 55, wherein the forces are given by at
least one of the group comprising:

$$\mathbf{F}_{\text{exc}} = \frac{(Z-n)e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r$$

$$\mathbf{F}_{\text{exc}} = \frac{(Z-(n-1))e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r$$

$$\mathbf{F}_{\text{mag}} = \frac{1}{4\pi\mu_0^2} \frac{1}{Z} \frac{\hbar^2}{m_e r^3} \sqrt{s(s+1)}$$

$$\mathbf{F}_{\text{diamagnetic}} = -\frac{\hbar^2}{4m_e r_1^2 r_1} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{diamagnetic}} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!} \frac{\hbar^2}{4m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{diamagnetic}} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!} \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{diamagnetic}} = -\left(\frac{2}{3} + \frac{2}{3} + \frac{1}{3}\right) \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = -\left(\frac{5}{3}\right) \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{diamagnetic 2}} = -\left[\frac{Z-3}{Z-2}\right] \frac{r_1 \hbar^2}{m_e r_3^4} 10\sqrt{3/4} \mathbf{i}_r$$

$$\mathbf{F}_{\text{diamagnetic 2}} = -\left[\frac{Z-n}{Z-(n-1)}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{m_e r_n^4} 10\sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{diamagnetic 2}} = -\left[\frac{Z-11}{Z-10}\right] \left(1 + \frac{\sqrt{2}}{2}\right) \frac{r_{10} \hbar^2}{m_e r_{11}^4} 10\sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{diamagnetic 2}} = -\left[\frac{Z-n}{Z-(n-1)}\right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{2}}{2} + \frac{1}{2}\right) \frac{r_{18} \hbar^2}{m_e r_n^4} 10\sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{diamagnetic 3}} = -\frac{1}{Z} \frac{8\hbar^2}{m_e r_{11}^3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{mag 2}} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{mag 2}} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{mag 2}} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_4} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{mag 2}} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{mag 2}} = (4+4+4) \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = \frac{1}{Z} \frac{12\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{mag 2}} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{\text{mag 2}} = \frac{1}{Z} \frac{8\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

57. (Previously Presented) The method of claim 55, wherein the radii are given by at least one of the group comprising:

$$r_1 = r_2 = a_0 \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right]$$

$$r_3 = r_4 = \frac{a_0 \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right) \pm a_0 \sqrt{\left(\frac{\left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)^2}{\left(Z-3 - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)} \right)^2 + 4 \left(\frac{\left(Z-3 \right) r_1}{Z-2} \right)^2}}}{2}$$

r₁ in units of a₀

$$r_n = \frac{\frac{a_0}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} \pm a_0}{2} \sqrt{\frac{1}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)^2} + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 \right)}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)}}$$

$$r_{10} = \frac{\frac{a_0}{\left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right)} \pm a_0}{2} \sqrt{\frac{1}{\left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right)^2} + \frac{20\sqrt{3} \left(\left[\frac{Z-10}{Z-9} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 \right)}{\left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right)}}$$

$$r_{11} = \frac{a_0 \left(1 + \frac{8}{Z} \sqrt{\frac{3}{4}} \right)}{(Z-10) - \frac{\sqrt{3}}{4r_{10}}} , \quad r_{10} \text{ in units of } a_0$$

$$\begin{aligned}
 r_{12} &= \frac{\frac{a_0}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)^{\pm a_0}} + \frac{1}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}^2} \\
 &\quad + \frac{20\sqrt{3}\left[\left[\frac{Z-12}{Z-11}\right]\left(1+\frac{\sqrt{2}}{2}\right)r_{10}\right]}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)^2} \\
 r_n &= \frac{\frac{a_0}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)^{\pm a_0}} + \frac{1}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}^2} \\
 &\quad + \frac{20\sqrt{3}\left[\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)r_{12}\right]}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)^2} \\
 r_m &= \frac{\frac{a_0\left(1+(C-D)\frac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)^{\pm a_0}} + \frac{\left(1+(C-D)\frac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)}^2} \\
 &\quad + \frac{20\sqrt{3}\left[\left[\frac{Z-n}{Z-(n-1)}\right]E_{r_m}\right]}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)^2}
 \end{aligned}$$

58. (Previously Presented) The method of claim 55, wherein the electric energy of each electron of radius r_n is given by at least one of the group comprising:

$$E(electric) = -\frac{(Z-(n-1))e^2}{8\pi\epsilon_0 r_s}$$

$$Ionization\ Energy(He) = -E(electric) + E(magnetic) \left(1 - \frac{1}{2} \left(\left(\frac{2}{3} \cos \frac{\pi}{3} \right)^2 + \alpha \right) \right)$$

$$Ionization\ Energy = -Electric\ Energy - \frac{1}{Z} Magnetic\ Energy$$

$$E(Ionization) = -Electric\ Energy - \frac{1}{Z} Magnetic\ Energy - E_r$$

$$E(ionization; Li) = \frac{(Z-2)e^2}{8\pi\epsilon_0 r_3} + \Delta E_{mag}$$

$$= 5.3178\ eV + 0.0860\ eV = 5.4038\ eV$$

$$E(ionization; B) = \frac{(Z-4)e^2}{8\pi\epsilon_0 r_3} + \Delta E_{mag}$$

$$= 8.147170901\ eV + 0.15548501\ eV = 8.30265592\ eV$$

$$E(ionization; Be) = \frac{(Z-3)e^2}{8\pi\epsilon_0 r_4} + \frac{2\pi\mu_0 e^2 h^2}{m_e^2 r_4^3} + \Delta E_{mag}$$

$$= 8.9216\ eV + 0.03226\ eV + 0.33040\ eV = 9.28430\ eV$$

$$E(ionization; Na) = -Electric\ Energy = \frac{(Z-10)e^2}{8\pi\epsilon_0 r_{11}} = 5.12592\ eV$$

59. (Previously Presented) The method of claim 55, wherein the radii of s electrons are given by

$$r_n = \frac{a_0 \left[\begin{array}{l} \left(1 + (C-D)\frac{\sqrt{3}}{2Z} \right)^2 \\ \left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right) \end{array} \right] \pm a_0 \left[\begin{array}{l} 20\sqrt{3} \left[\left(\frac{Z-n}{Z-(n-1)} \right) E r_m \right] \\ \left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right) \end{array} \right]}{2}$$

r_m in units of a₀

where positive root must be taken in order that r_n > 0;

Z is the nuclear charge, n is the number of electrons, r_m is the radius of the proceeding filled shell(s) given by

$$r_n = \frac{a_0 \left(1 + (C - D) \frac{\sqrt{3}}{2Z} \right)^{\pm a_0} \left[\begin{array}{l} \left(1 + (C - D) \frac{\sqrt{3}}{2Z} \right)^2 \\ \left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right) \end{array} \right]^{20\sqrt{3}} \left[\frac{Z-n}{Z-(n-1)} \right] E_{r_m}}{2 \left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)}$$

r_m in units of a_0

for the preceding s shells(s);

$$r_n = \frac{a_0 \left(1 + (C - D) \frac{\sqrt{3}}{2Z} \right)^{\pm a_0} \left[\begin{array}{l} 1 \\ \left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right) \end{array} \right]^{20\sqrt{3}} \left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3}{2 \left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)}$$

r_3 in units of a_0

for the 2p shells, and

$$r_a \approx \frac{\left(\frac{a_0}{(Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}}} \right)^{\pm a_0} - \frac{1}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)} + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2} \right) r_{12} \right)}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)}}{2}$$

r₁₂ in units of a₀

for the 3p shell;

the parameter A corresponds to the diamagnetic force, $F_{diamagnetic}$:

$$\mathbf{F}_{diamagnetic} = -\frac{\hbar^2}{4m_e r_1^2 r_1} \sqrt{s(s+1)} \mathbf{i}_r$$

the parameter B corresponds to the paramagnetic force, F_{mag2} :

$$\mathbf{F}_{mag2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_1^2 r_4} \sqrt{s(s+1)} \mathbf{i}_r$$

the parameter C corresponds to the diamagnetic force, $F_{diamagnetic\ 3}$:

$$\mathbf{F}_{diamagnetic\ 3} = -\frac{1}{Z} \frac{8\hbar^2}{m_e r_{11}^3} \sqrt{s(s+1)} \mathbf{i}_r$$

the parameter D corresponds to the paramagnetic force, F_{mag} :

$$\mathbf{F}_{mag} = \frac{1}{4m_e^2} \frac{1}{Z} \frac{\hbar^2}{m_e r^3} \sqrt{s(s+1)}$$

, and

the parameter E corresponds to the diamagnetic force, $F_{diamagnetic\ 2}$ due to a

relativistic effect with an electric field for $r > r_n$:

$$\begin{aligned}\mathbf{F}_{diamagnetic\ 2} &= -\left[\frac{Z-3}{Z-2}\right] \frac{r_1 \hbar^2}{m_e r_3^4} 10\sqrt{3/4} \mathbf{i}_r \\ \mathbf{F}_{diamagnetic\ 2} &= -\left[\frac{Z-11}{Z-10}\right] \left(1 + \frac{\sqrt{2}}{2}\right) \frac{r_{10} \hbar^2}{m_e r_{11}^4} 10\sqrt{s(s+1)} \mathbf{i}_r, \text{ and} \\ \mathbf{F}_{diamagnetic\ 2} &= -\left[\frac{Z-n}{Z-(n-1)}\right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{2}}{2} + \frac{1}{2}\right) \frac{r_{1s} \hbar^2}{m_e r_n^4} 10\sqrt{s(s+1)} \mathbf{i}_r\end{aligned}$$

wherein the parameters of atoms filling the 1s, 2s, 3s, and 4s orbitals are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of s Electrons (s state)	Dia e	Para e	Dia mag	Para mag	Dia g.	Para E	Dia Force Factor
				A	B	C	D			
Neutral 1s ¹ 1 e Atom		$^2S_{1/2}$	$\frac{\uparrow}{1s}$	0	0	0	0	0		
<i>H</i>										
Neutral 1s ² 2 e Atom		1S_0	$\frac{\uparrow\downarrow}{1s}$	0	0	0	1	0		
<i>He</i>										
Neutral 2s ¹ 3 e Atom		$^2S_{1/2}$	$\frac{\uparrow}{2s}$	1	0	0	0	0		
<i>Li</i>										
Neutral 2s ² 4 e Atom		1S_0	$\frac{\uparrow\downarrow}{2s}$	1	0	0	1	0		
<i>Be</i>										
Neutral 1s ² 2s ² 2p ⁶ 3s ¹ 11 e Atom		$^2S_{1/2}$	$\frac{\uparrow}{3s}$	1	0	8	0	0		
<i>Na</i>										
Neutral 1s ² 2s ² 2p ⁶ 3s ² 12 e Atom		1S_0	$\frac{\uparrow\downarrow}{3s}$	1	3	12	1	0		
<i>Mg</i>										
Neutral 1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ¹ 19 e Atom		$^2S_{1/2}$	$\frac{\uparrow}{4s}$	2	0	12	0	0		
<i>K</i>										
Neutral 1s ² 2s ² 2p ⁶ 3s ² 3p ⁶ 4s ² 20 e Atom		1S_0	$\frac{\uparrow\downarrow}{4s}$	1	3	24	1	0		
<i>Ca</i>										
1 e ion 1s ¹		$^2S_{1/2}$	$\frac{\uparrow}{1s}$	0	0	0	0	0		
2 e ion 1s ²		1S_0	$\frac{\uparrow\downarrow}{1s}$	0	0	0	1	0		
3 e ion 2s ¹		$^2S_{1/2}$	$\frac{\uparrow}{-}$	-	-	-	-	-		

4 e ion	$2s^2$	1S_0	$\frac{\uparrow \downarrow}{2s}$	1	0	0	1	1
11 e ion	$1s^2 2s^2 2p^6 3s^1$	${}^2S_{1/2}$	$\frac{\uparrow}{3s}$	1	4	8	0	$1 + \frac{\sqrt{2}}{2}$
12 e ion	$1s^2 2s^2 2p^6 3s^2$	1S_0	$\frac{\uparrow \downarrow}{3s}$	1	6	0	0	$1 + \frac{\sqrt{2}}{2}$
19 e ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$	${}^2S_{1/2}$	$\frac{\uparrow}{4s}$	3	0	24	0	$2 - \sqrt{2}$
20 e ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$	1S_0	$\frac{\uparrow \downarrow}{4s}$	2	0	24	0	$2 - \sqrt{2}$

60. (Original) The method of claim 59, with the radii, r_n , wherein the ionization energy for atoms having an outer s-shell are given by the negative of the electric energy, $E(electric)$, given by:

$$E(Ionization) = -Electric\ Energy = \frac{(Z - (n - 1))e^2}{8\pi\epsilon_0 r_n}$$

except that minor corrections due to the magnetic energy must be included in cases wherein the s electron does not couple to p electrons as given by

$$Ionization\ Energy(He) = -E(electric) + E(magnetic) \left(1 - \frac{1}{2} \left(\left(\frac{2}{3} \cos \frac{\pi}{3} \right)^2 + \alpha \right) \right)$$

$$Ionization\ Energy = -Electric\ Energy - \frac{1}{Z} Magnetic\ Energy$$

$$E(Ionization; Li) = \frac{(Z - 2)e^2}{8\pi\epsilon_0 r_3} + \Delta E_{mag}$$

$$= 5.3178\ eV + 0.0860\ eV = 5.4038\ eV$$

$$E(Ionization) = E(Electric) + E_T$$

$$E(Ionization; Be) = \frac{(Z - 3)e^2}{8\pi\epsilon_0 r_4} + \frac{2\pi\mu_0 e^2 h^2}{m_e^2 r_4^3} + \Delta E_{mag}$$

and

$$= 8.9216 \text{ eV} - 0.03226 \text{ eV} + 0.33040 \text{ eV} = 9.28430 \text{ eV}$$

$$E(\text{Ionization}) = -\text{Electric Energy} - \frac{1}{Z} \text{Magnetic Energy} - E_r.$$

61. (Previously Presented) The method of claim 55, wherein the radii and energies of the 2p electrons are solved using the forces given by

$$\mathbf{F}_{el} = \frac{(Z-n)e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!} \frac{\hbar^3}{4m_e r_n^2 r_1} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 1} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 3} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{m_e r_n^4} 10 \sqrt{s(s+1)} \mathbf{i}_r$$

and the radii r_3 are given by

$$r_4 = r_3 = \frac{\pm a_0 \sqrt{\left(\frac{Z-3}{Z-2} \right) r_i 10 \sqrt{\frac{3}{4}}} + 4 \left(\frac{(Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \sqrt{\frac{3}{4}}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \sqrt{\frac{3}{4}} \right)^2 + 4 \left(\frac{(Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \sqrt{\frac{3}{4}}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \sqrt{\frac{3}{4}} \right)} \right)}}{2}$$

r_i in units of a_0

62. (Original) The method of claim 61, wherein the electric energy given by

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z - (n-1))e^2}{8\pi\epsilon_0 r_a}$$

gives the corresponding ionization energies.

63. (Previously Presented) The method of claim 55, wherein for each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^{n-4}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_0 \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right]$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by

$$r_4 = r_3 = \frac{\left(a_0 \left(1 - \frac{\sqrt{3}}{4} \right) \right)}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{4} \right)} \pm a_0 \sqrt{\frac{\left(\left(1 - \frac{\sqrt{3}}{4} \right)^2 + 4 \right) \left[\frac{Z-3}{Z-2} \right] r_1^{10} \sqrt{\frac{3}{4}}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{4} \right)^2}}$$

r_1 in units of a_0

and n-4 electrons in an orbitsphere with radius r_n given by

$$r_n = \frac{\left(a_0 \left(\left(Z - (n-1) \right) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right) \right) \pm a_0 \sqrt{\frac{1}{\left(\left(Z - (n-1) \right) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)^2} + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 \right)^2}}}{2}$$

r_3 in units of a_0

the positive root must be taken in order that $r_n > 0$;

the parameter A corresponds to the diamagnetic force, $F_{diamagnetic}$:

$$\mathbf{F}_{diamagnetic} = - \sum_m \frac{(\ell + |m|)}{(2\ell + 1)(\ell - |m|)} \frac{\hbar^2}{4m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

and the parameter B corresponds to the paramagnetic force, F_{mag2} :

$$\mathbf{F}_{mag1} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r,$$

$$\mathbf{F}_{mag2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{mag2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

wherein the parameters of five through ten-electron atoms are

Atom Type	Electron Configuration n	Ground Orbital State Term of Arrangement of 2p Electrons (2p state)	Diamagnetism c Force Factor r A	Paramagnetism etic Force Factor r B
Neutral 5 e Atom <i>B</i>	$1s^2 2s^2 2p^1$	${}^2P_{1/2}^0$ $\begin{array}{ccc} \uparrow & & \\ -1 & 0 & -1 \end{array}$	2	0
Neutral 6 e Atom <i>C</i>	$1s^2 2s^2 2p^2$	3P_0 $\begin{array}{ccc} \uparrow & \uparrow & \\ -1 & 0 & -1 \end{array}$	$\frac{2}{3}$	0
Neutral 7 e Atom <i>N</i>	$1s^2 2s^2 2p^3$	${}^4S_{3/2}^0$ $\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ -1 & 0 & -1 \end{array}$	$\frac{1}{3}$	1
Neutral 8 e Atom <i>O</i>	$1s^2 2s^2 2p^4$	1P_1 $\begin{array}{ccc} \uparrow \downarrow & \uparrow & \uparrow \\ -1 & 0 & -1 \end{array}$	1	2
Neutral 9 e Atom <i>F</i>	$1s^2 2s^2 2p^5$	${}^2P_{3/2}^0$ $\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \\ -1 & 0 & -1 \end{array}$	$\frac{2}{3}$	3
Neutral 10 e Atom <i>Ne</i>	$1s^2 2s^2 2p^6$	1S_0 $\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \downarrow \\ -1 & 0 & -1 \end{array}$	0	3
5 e Ion	$1s^2 2s^2 2p^1$	${}^2P_{1/2}^0$ $\begin{array}{ccc} \uparrow & & \\ -1 & 0 & -1 \end{array}$	$\frac{5}{3}$	1
6 e Ion	$1s^2 2s^2 2p^2$	3P_0 $\begin{array}{ccc} \uparrow & \uparrow & \\ -1 & 0 & -1 \end{array}$	$\frac{5}{3}$	4
7 e Ion	$1s^2 2s^2 2p^3$	${}^4S_{3/2}^0$ $\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ -1 & 0 & -1 \end{array}$	$\frac{5}{3}$	6
8 e Ion	$1s^2 2s^2 2p^4$	3P_1 $\begin{array}{ccc} \uparrow \downarrow & \uparrow & \uparrow \\ -1 & 0 & -1 \end{array}$	$\frac{5}{3}$	6
9 e Ion	$1s^2 2s^2 2p^5$	${}^2P_{3/2}^0$ $\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \\ -1 & 0 & -1 \end{array}$	$\frac{5}{3}$	9
10 e Ion	$1s^2 2s^2 2p^6$	1S_0 $\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \downarrow \\ -1 & 0 & -1 \end{array}$	$\frac{5}{3}$	12

64. (Original) The method of claim 63, wherein the ionization energy for the boron atom is given by

$$E(\text{ionization}; B) = \frac{(Z-4)e^2}{8\pi\epsilon_0 r_s} + \Delta E_{\text{mag}} \\ = 8.147170901 \text{ eV} + 0.15548501 \text{ eV} = 8.30265592 \text{ eV}$$

65. (Original) The method of claim 63, wherein the ionization energies for the n-electron atoms having the radii, r_n , are given by the negative of the electric energy, $E(\text{electric})$, given by

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z-(n-1))e^2}{8\pi\epsilon_0 r_n}$$

66. (Previously Presented) The method of claim 55, wherein the radii of the 3p electrons are given using the forces given by

$$\mathbf{F}_{\text{elec}} = \frac{(Z-n)e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r \\ \mathbf{F}_{\text{diamagnetic}} = -\sum_m \frac{(\ell+|m|)}{(2\ell+1)(\ell-|m|)} \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{\text{diamagnetic}} = -\left(\frac{2}{3} + \frac{2}{3} + \frac{1}{3}\right) \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = -\left(\frac{5}{3}\right) \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{\text{mag}2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{\text{mag}2} = (4+4+4) \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = \frac{1}{Z} \frac{12\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{\text{mag}2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{\text{mag}2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{\text{mag}2} = \frac{1}{Z} \frac{8\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

and the radii r_{12} are given by

$$r_{12} = \frac{a_0}{2} \left[\sqrt{\left(\frac{1}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)} \right)^2 + \frac{20\sqrt{3} \left(\left[\frac{Z-12}{Z-11} \right] \left(1 + \frac{\sqrt{2}}{2} \right) r_{10} \right)}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)}} \right]$$

r₁₀ in units of a₀

67. (Original) The method of claim 66, wherein the ionization energies are given by electric energy given by:

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z - (n - 1))e^2}{8\pi\varepsilon_0 r_n}$$

68. (Previously Presented) The method of claim 55, wherein for each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^6 3s^2 3p^{n-12}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_0 \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right]$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\pm a_0 \sqrt{\left(\frac{Z-3}{Z-2} \right) r_1 + 4 \left(\frac{Z-3}{Z-2} - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1} \right)^2}}{2}$$

r_1 in units of a_0

three sets of paired indistinguishable electrons in an orbitsphere with radius r_{10} given by:

$$r_{10} = \frac{\pm a_0 \sqrt{\left(\frac{Z-9}{Z-24} - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right)^2 + 4 \left(\frac{Z-10}{Z-9} \left(1 - \frac{\sqrt{2}}{2} \right) r_3 + \left(\frac{Z-9}{Z-24} - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right) \right)^2}}{2}$$

r_3 in units of a_0

two indistinguishable spin-paired electrons in an orbitsphere with radius r_{12} given by:

$$r_{12} = \frac{\frac{a_0}{(Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}\right)^2 + \frac{20\sqrt{3}\left[\left\lfloor \frac{Z-12}{Z-11} \right\rfloor\left(1+\frac{\sqrt{2}}{2}\right)r_{10}\right]}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}}}{2}$$

r₁₀ in units of a₀

and n-12 electrons in a 3p orbitsphere with radius r_n given by

$$r_n = \frac{\frac{a_0}{(Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}\right)^2 + \frac{20\sqrt{3}\left[\left\lfloor \frac{Z-n}{Z-(n-1)} \right\rfloor\left(1-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)r_{12}\right]}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}}}{2}$$

r₁₂ in units of a₀

where the positive root must be taken in order that r_n > 0;

the parameter A corresponds to the diamagnetic force, F_{diamagnetic}:

$$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!} \frac{\hbar^2}{4m\epsilon_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_t$$

and the parameter B corresponds to the paramagnetic force, F_{mag2}:

$$\begin{aligned}\mathbf{F}_{mag\ 2} &= \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{mag\ 2} &= (4+4+4) \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = \frac{1}{Z} \frac{12\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{mag\ 2} &= \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r \\ \mathbf{F}_{mag\ 2} &= \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r, \text{ and} \\ \mathbf{F}_{mag\ 2} &= \frac{1}{Z} \frac{8\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r,\end{aligned}$$

wherein the parameters of thirteen to eighteen-electron atoms are

Atom Type	Electron Configuration	Ground State Orbital Term	Orbital Arrangement of 3p Electrons (3p state)	Diamagnetic Force Factor A	Paramagnetic Force Factor B
Neutral 13 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^1$	${}^2P_{1/2}^0$	$\frac{\uparrow}{1} \quad \frac{-}{0} \quad \frac{-}{-1}$	$\frac{11}{3}$	0
<i>Al</i>					
Neutral 14 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^2$	3P_0	$\frac{\uparrow}{1} \quad \frac{\uparrow}{0} \quad \frac{-}{-1}$	$\frac{7}{3}$	0
<i>Si</i>					
Neutral 15 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^3$	${}^4S_{3/2}^0$	$\frac{\uparrow}{1} \quad \frac{\uparrow}{0} \quad \frac{\uparrow}{-1}$	$\frac{5}{3}$	2
<i>P</i>					
Neutral 16 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^4$	3P_2	$\frac{\uparrow\downarrow}{1} \quad \frac{\uparrow}{0} \quad \frac{\uparrow}{-1}$	$\frac{4}{3}$	1
<i>S</i>					
Neutral 17 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^5$	${}^2P_{3/2}^0$	$\frac{\uparrow\downarrow}{1} \quad \frac{\uparrow\downarrow}{0} \quad \frac{\uparrow}{-1}$	$\frac{2}{3}$	2
<i>Cl</i>					
Neutral 18 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^6$	1S_0	$\frac{\uparrow\downarrow}{1} \quad \frac{\uparrow\downarrow}{0} \quad \frac{\uparrow\downarrow}{-1}$	$\frac{1}{3}$	4
<i>Ar</i>					
13 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^1$	${}^2P_{1/2}^0$	$\frac{\uparrow}{1} \quad \frac{-}{0} \quad \frac{-}{-1}$	$\frac{5}{3}$	12
14 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^2$	3P_0	$\frac{\uparrow}{1} \quad \frac{\uparrow}{0} \quad \frac{-}{-1}$	$\frac{1}{3}$	16
15 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^3$	${}^4S_{3/2}^0$	$\frac{\uparrow}{1} \quad \frac{\uparrow}{0} \quad \frac{\uparrow}{-1}$	0	24
16 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^4$	3P_2	$\frac{\uparrow\downarrow}{1} \quad \frac{\uparrow}{0} \quad \frac{\uparrow}{-1}$	$\frac{1}{3}$	24
17 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^5$	${}^2P_{3/2}^0$	$\frac{\uparrow\downarrow}{1} \quad \frac{\uparrow\downarrow}{0} \quad \frac{\uparrow}{-1}$	$\frac{2}{3}$	32
18 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6$	1S_0	$\frac{\uparrow\downarrow}{1} \quad \frac{\uparrow\downarrow}{0} \quad \frac{\uparrow\downarrow}{-1}$	0	40

69. (Original) The method of claim 68 wherein the ionization energies for the n-electron 3p atoms are given by electric energy given by:

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z - (n-1))e^2}{8\pi\epsilon_0 r_n}$$

70. (Original) The method of claim 68 wherein the ionization energy for the aluminum atom is given by

$$\begin{aligned} E(\text{ionization; Al}) &= \frac{(Z-12)e^2}{8\pi\epsilon_0 r_{13}} + \Delta E_{\text{mag}} \\ &= 5.95270 \text{ eV} + 0.031315 \text{ eV} = 5.98402 \text{ eV} \end{aligned}$$